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Future Developments in the Control of Power Systems

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VII. POWER SYSTEM CONTROL

Future developments in the control of power systems

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The basic requirement of the control system is to ensure the supply of electric power at constant voltage and frequency exactly in accordance with the instantaneous demand. Its design is dominated by (*a*) the characteristics of the consumers' demand; (*b*) the provision of a reliable supply, even under fault conditions, at minimum cost; (*c*) the technical and operational characteristics of a mixture of fossil-fired, nuclear and hydro generators; (*d*) an assessment of the benefits to be gained and the cost of implementation. Large modern generating units are complex and their characteristics and those of the transmission system are such as to increase the difficulties of operation and control beyond the unaided capabilities of human operators. Automatic aids, many of them computer-based, are being used increasingly. The paper reviews the present position and indicates how advances in control technology could lead to on-line control of the system at acceptable cost.

Introduction

The history of the control and operation of modern power systems is one of evolution to meet the changing characteristics of the plant and the increasing dependence of modern society on a cheap, reliable supply of electric power. Within the 30 to 40 years useful lifetime of a generating unit the capacity of the highest merit plant has increased by a factor of about 10 and plant which was initially conceived for base-load drops successively in merit order until it becomes useful in peak load conditions only and finally is no longer economic to retain. The operation and control has to be adapted during its life to enable it to fulfil its changing role in the system.

The role of the main interconnected transmission system has also changed. Initially required to reinforce the local networks in the event of emergency, with the siting of new large power stations on the coal fields for economy reasons, it is now required for bulk transmission to the main load centres, and these trends will continue with the siting of large oil and nuclear stations on the coast away from the main load centres. To take advantage of this siting policy, grid control has to optimize the scheduling of load among the available generators consistent with providing a secure supply.

Thus control of a modern power system has to be seen as a complex integrated operation involving all types of plant and sufficiently flexible to accommodate new plant, the changing roles of earlier plant and the changing patterns of consumer demand. Much academic effort is being devoted to the theory of control of large systems and to the techniques of implementation, but in the words of Nagel (1972) at a recent conference 'we have to strike some middle ground between the practical constraints of real power systems and the more esoteric and undoubtedly fascinating world of system control and automation'. In this paper we explore this middle ground in some detail.

Consumer demand

The first requirement in considering the design of a control system is a statement on the characteristics of the demand. Since no significant storage of alternating current (a.c.) electrical energy is available, the consumers' demand has to be met precisely at all times. It consists essentially of the sum of a very large number of individual loads varying in size up to several megawatts, switched on and off in a stochastic manner, but is often regarded as a long-term component which can be predicted to some accuracy, together with a random short-term component. The predictable part will have a daily component rising to a maximum during the day and falling to a minimum in the early hours of the morning. It will vary with time of year, day of the week, weather conditions and over very long periods with the economic growth rate of the community. A considerable amount of work has been and is being done in perfecting prediction methods and accuracies of a few per cent are achieved over prediction periods of several hours. Figure 1 shows typical daily variations in the C.E.G.B. (England and Wales) demand which are typical of most power systems. It will be noted that it ranges over a factor of about two during a 24 h period and the maximum rate of change is about 10 GW/h.

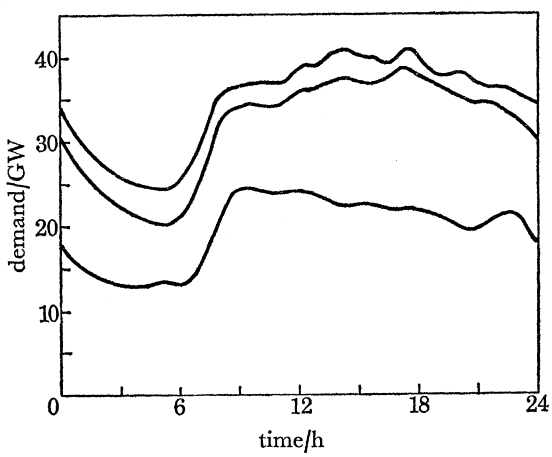

Figure 1. Daily load variations in C.E.G.B. system (1971–2).

The properties of the short-term random component are much more difficult to quantify. Measurements on the C.E.G.B. system have shown that, at night when the demand was reasonably constant (about 16 GW) for several hours, the mean square variation between the demand at time t and that at time $t+\tau$ was linear with τ and rose to 10^4 $(\mathrm{MW})^2$ in 30 min. This is characteristic of a random walk process. Similar results were obtained for the daytime demand using the previous day's record to remove the long-term trend but the mean square variation depended on time of day and could be as high as 6×10^4 $(\mathrm{MW})^2$ in 30 min, when the total demand was 30 GW. Work is continuing at C.E.R.L. to obtain a more useful representation.

The power system

To meet the demand a power company operates a number of generators which are connected to the consumers' terminals by a transmission system. The generators, by which term we shall denote the whole installation required to convert from a primary source of energy to electrical energy, are of four main types, fossil-fired (coal, gas or oil), nuclear, hydro and gas turbines.

They generate alternating current and because of the close coupling provided by the transmission system can be assumed to run synchronously.

The main transmission network is usually meshed, i.e. there is always more than one path between any node and any other node to provide a measure of security. Generators are connected to some nodes. Most nodes provide a spur to a secondary or distribution network at lower voltage to feed consumer terminals and in general the distribution networks do not bridge nodes on the main transmission network. In most systems links are provided to other systems. If these links carry alternating current a problem arises in synchronizing the two systems since relatively small power imbalances in the two systems can lead to unacceptably large flows in the interconnexions.

Different power systems have different plant characteristics and different control requirements. In fact, for control purposes, the characteristics of the C.E.G.B. system are markedly different from those of other countries in a number of respects. First, the units are mainly 'thermal', i.e. the primary fuel (coal, oil, gas, nuclear) is used to raise steam to drive the turbo-alternators and there is little hydro plant. To provide protection against the loss of output of a large generator all power systems have to ensure that they have adequate running spare, i.e. plant which can increase load in a matter of seconds. This is much more difficult to provide with thermal than with hydro plant. Secondly, apart from the connexion with Scotland, a relatively much smaller system, there are no a.c. ties with any other system. Thus there is no requirement to maintain a common frequency standard with other systems and the U.K. system has its own frequency. There are, of course, limits within which this frequency must be maintained but they do not constitute such a severe control requirement. Thirdly, the transmission network is heavily meshed with no long unbroken lines carrying power from a generator. If a generator is connected to a system by a long line its rotor can be made to oscillate about its mean position by a system distrubance and if the oscillation is large enough it may lose synchronism. This form of instability has been largely eliminated from the C.E.G.B. system partly because the system is so compact, partly by careful siting of the generators and partly by operating the system according to procedures defined by comprehensive studies in the planning phase. However, the heavily meshed network does increase the currents which can flow under short-circuit fault conditions. Finally, the C.E.G.B. system is a very large, probably the largest, system under unified control and the control problem has a correspondingly large number of variables. For the remainder of this paper emphasis will be placed on the C.E.G.B. system.

Control of thermal generators

The energy conversion process in a thermal generator is shown in figure 2 which represents a single generator feeding a load. The control problem can be demonstrated by considering the effects of a sudden step increase ΔP in demand P_0. The effect is to reduce the kinetic energy stored in the turbo-alternator and the voltage of the supply. The excitation control system reacts rapidly to restore the voltage and the loss of kinetic energy causes an initial reduction in speed of rotation and hence frequency. A conventional governor having proportional action attempts to restore frequency by opening the steam valve and admitting more steam and the ratio $\Delta f/f_0$ to $\Delta P/P_0$ is termed the 'incremental droop'. The time constant associated with this action is a few seconds. The increased energy flow provided by the governor is now drawn from the reserves in the boiler and these fall at a rate measured by the fall in steam pressure. In a normal

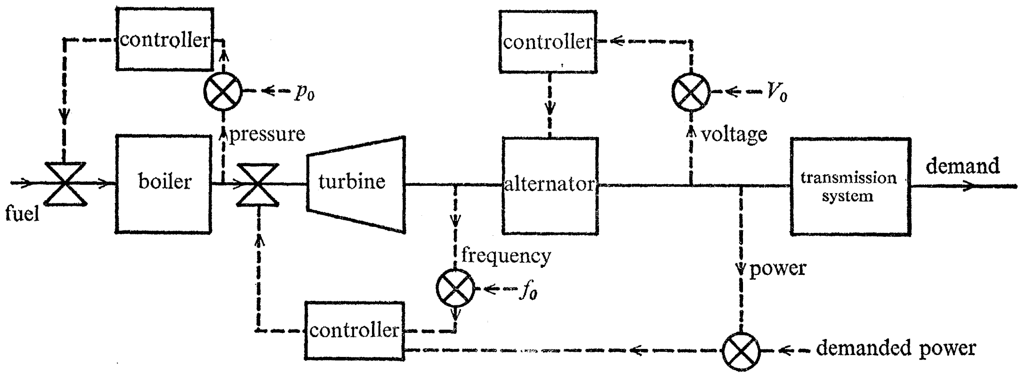

FIGURE 2. Schematic diagram of generator.

boiler-following-turbine control system the pressure signal is used to increase the firing rate in the boiler to maintain an overall energy balance, the time constant of this process being several minutes. The resulting MW output is shown in figure 3. It is clear that because of the lags in the boiler the response of the unit to the sudden change in demand is inadequate. Similar arguments apply if the unit is asked to increase load at a steady rate. In practice the pressure deviations are limited by an upper bound when the safety valves blow and by a lower bound set by the pressure unloading gear which trips off the set and these are typically $\pm 10\%$. This limits the rate at which the load on the unit can be changed in practice.

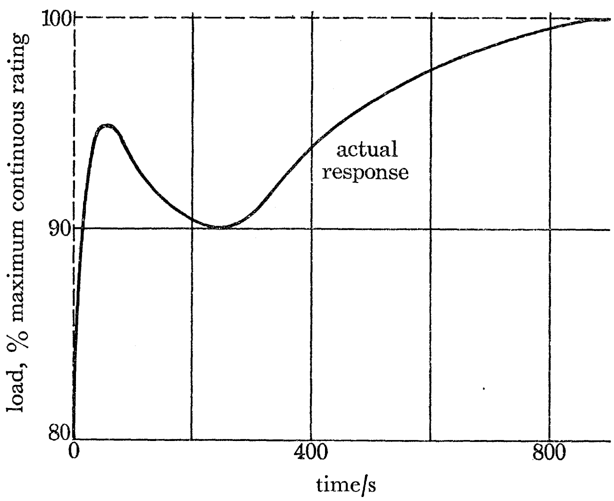

FIGURE 3. Typical response to step increase in demand.

Another factor which limits the rate of loading is the thermal stresses which arise in the thick pressurized parts of the turbine, particularly the casing, and in the boiler tubes. Fracture stress will set a limit to the loading rates and amplitude. The thermal stress also gives rise to fatigue strains if many load changes are required during the life of the unit and, although it is not yet possible to quantify the effect precisely, upper limits can be set which must constrain the operation of the generator. Similar constraints apply to the fuel of nuclear reactors.

The current to be supplied by the generator is not always in phase with the voltage. Using the conventional complex notation the power output of the units can be split into real and imaginary, or reactive, power. At any given real power output there are limits to the reactive power which can be supplied, set, on the one hand, by the fact that the rotor angle enters the unstable region and on the other by over-heating of the alternator. These limits must be observed in the operation of the generator. There are thus a number of constraints which the characteristics of the generator impose on the loading profile it can be asked to follow.

Automatic controls are supplied on large units for voltage regulation, frequency regulation, boiler control and sequence control of start-up and shut-down. Comprehensive mathematical models have beeen formulated to describe the dynamics of a modern generator to enable control studies to be made. These models run on large digital computers (IBM 370) in around real time and on a large hybrid installation in one-tenth of real time.

Transmission system

The main transmission system can be regarded as a large set of nodes connected by transmission lines all operating at the same nominal voltage. The equations relating the real and reactive power flows into a node to the voltage and phase angle, i.e. the phase of the voltage relative to some reference point in the network, can be written to a sufficiently high accuracy in the form:

$$\begin{bmatrix}\Delta \boldsymbol{P}\\ \Delta \boldsymbol{Q}\end{bmatrix} = [J]\begin{bmatrix}\Delta \boldsymbol{\theta}\\ \Delta \boldsymbol{V}\end{bmatrix}, \qquad [J] = \begin{bmatrix}\partial \boldsymbol{P}/\partial \boldsymbol{\theta} & \partial \boldsymbol{P}/\partial \boldsymbol{V}\\ \partial \boldsymbol{Q}/\partial \boldsymbol{\theta} & \partial \boldsymbol{Q}/\partial \boldsymbol{V}\end{bmatrix},$$

where $\Delta \boldsymbol{P}$, $\Delta \boldsymbol{Q}$, $\Delta \boldsymbol{\theta}$, $\Delta \boldsymbol{V}$ are the n-vectors of real power injection, reactive power injection, phase angle and voltage referred to a nominal operating state and n is the number of nodes. The real power flow in the lines can be deduced from a knowledge of $\boldsymbol{\theta}_0$, $\boldsymbol{V}_0$, $\Delta \boldsymbol{\theta}$, $\Delta \boldsymbol{V}$. Thus to estimate the power flows in the lines involves the inversion of the matrix $[J]$ and for a large system this is a formidable computing problem. An estimate of the real power flows is required when determining the generation pattern since there is a limit to the loading of a line, set by the loss of mechanical strength of the conductors due to ohmic heating. An operational requirement is that the line limits should not be exceeded, not only under normal conditions but also in the event of any single line, or double circuit line (i.e. two lines carried on the same towers) being disconnected due to a fault. This requirement is known as transmission security and is an important factor in the operation of the system. An automatic protection system is provided for all lines so that in the event of excessive current flows, due perhaps to faults on the system, circuit breakers are opened automatically to protect the line.

Voltage control is exercised by adjustment of reactive power injection, transformer tappings and various forms of reactive compensation. In practice it is found that these latter adjustments are required relatively infrequently so that they can be determined by off-line studies on large computers. To make this possible it is important that the construction and operation of the system be planned carefully in advance.

System control

The chief requirement of the control system is to meet the demands for electric power of the consumers as economically as possible and with a high level of security. When the changes in demand with time are relatively small they can be met by the governors of those generators which are sensitive to frequency changes. However, as we have seen, for larger changes the steam pressure in the generators may deviate outside acceptable limits and since the response of the boilers is relatively slow, it is necessary to predict the larger changes in load so as to prepare the firing of the boilers. Moreover, since the operating costs of the generators vary over a wide range, governor control is not necessarily the most economic way of satisfying the demand and neither is it necessarily secure in that it may reduce the spinning spare below the acceptable value and also the resulting power flows in the transmission system may not satisfy the security requirements. Thus the predictable component of the demand is apportioned to the generators according to an economic schedule calculated in advance; the small random component which cannot be predicted is met by goveror control.

The calculation of the economic schedule is an optimization problem of some complexity. It can be formulated as follows

The generating costs have to be minimized subject to a number of constraints. The most important of these are:

(*a*) The predicted demand should be met.

(*b*) The generator loads are either zero or must lie between upper and lower limits which vary according to the operating conditions of the generators.

(*c*) Loading rates must be restricted by plant conditions.

(*d*) The reactive power requirements must be satisfied.

(*e*) Sufficient spare must be carried to cover prediction errors and the possible loss of the largest unit.

(*f*) The requirements for transmission security, as defined earlier, must be satisfied.

In general the problem is nonlinear and techniques for solving it, for the size of system we are considering, are only just becoming available and they require considerable computing power. In general, the solution is obtained by off-line computers which define an operating schedule for several hours ahead and appropriate action to meet foreseen contingencies. In the event there are changes in the transmission configuration and in the availability of generators which can invalidate the schedules. Short-term action has to be taken by Grid Control but modern large generating units are proving less flexible than earlier plant and it is becoming increasingly necessary to improve the speed of response of the control system. The need is for a computing system of adequate computing power receiving data on-line from the power system, and for control systems within the power stations capable of following its instructions.

An experiment in automatic control

The need for close on-line control of the complete generation system introduces the possibility of fully automatic control. Some 8 years ago an experiment was conducted in a selected area of the southwest of England to determine to what extent automatic control is feasible technically and operationally, what problems arise and what are the likely benefits (Farmer, James, Moran & Pettit 1969). The area chosen consisted of 32 generators with a total capacity of 1800 MW and their associated transmission network and load points, and it could be operated either as part of the national system or, at certain times, isolated from the national system. The test system was controlled by a central computer installed in the Bristol Grid Control Centre. It received from the Board's standard telemetry system indications of network switching, power flows in lines and generator outputs and was fed by the operator with information on generator constraints (e.g. maximum and minimum loads, maximum rates of loading, spare capability), generating costs and required transfers to and from the main system. The primary functions of the computer were to:

(*a*) Accept these data.

(*b*) Use them to predict the total demand and the demand at the various feed points to the consumers for a fixed period ahead, usually 10 min.

(*c*) Use the prediction to obtain an economic, secure schedule.

(*d*) Dispatch the target loads to the generators via a telemetry link.

(*e*) Drive c.r.t. displays to enable the operator to monitor the schedules and if necessary intervene.

(*f*) Analyse the results.

Each generator was supplied with a specially designed controller (Moran, Bain & Sohal 1968) to enable it to accept the target and to drive the MW output to meet it automatically and at the same time respond in a prescribed manner to variations in system frequency.

In the design of the experiment a number of important simplifying assumptions were made. The determination of which generators should run (an operation known as ordering or, in the U.S.A., committing) was a separate operation not carried out by the computer whose function was restricted to specifying the load on those generators which were actually connected. It is now generally accepted that much of the economic advantage offered by computer control of a power system lies in the accurate determination of a secure, economic ordering schedule. It was also assumed that the Board's telemetry system would be used as far as possible since specially provided telemetry would be very expensive. When difficulties arose because of a telemetry failure the operator was relied upon to provide the missing information. A third important assumption was that the largely manual control of the boiler systems at the power stations would be adequate. To attempt to provide automatic boiler control would have entailed considerable technical development, probably peculiar to each station, and would have been prohibitively expensive. The effect of this was to restrict very considerably the rate of loading which the generators could achieve in practice for large load changes.

As might be expected from such a complex experiment in which many of the parameters could not be tightly controlled, e.g. consumers' demand, availability and performance of generators, telemetry faults, etc., the results were not entirely conclusive. It was, however, demonstrated that, within the constraints of the experiment, automatic control of a power system of the size is technically feasible and that the standard of control was higher. One of the most important features of any control system is its stability when subjected to external disturbances. A theoretical assessment was made (Farmer 1968) to determine the range of parameters over which the system would be stable and a limited number of experiments when the test system was isolated confirmed the assessment.

A number of technical problems were emphasized or exposed. The design and implementation of the computer software for such a complex control system is difficult. The quantities of data were large and had to be stored in a well-defined manner so that they could be updated quickly, their interrelation was clear and they could be readily accessed either for display or for use in algorithms. The computer's executive had to be capable of handling a wide range of jobs of widely differing lengths and data requirements and with as fast a response as possible. Operators' requests for information had to be satisfied in a second or two and some of these, for example a check on transmission security, involved lengthy computation. It was also often necessary to modify programs, particularly those associated with the c.r.t. displays and other output devices, with minimum risk to data files and other programs.

In the event the telemetry system, although adequate for conventional manual control, proved insufficiently reliable. False data on line flows, generator outputs and network switch positions sometimes gave rise to unacceptable schedules and manual intervention was necessary to correct the data. Algorithms were incorporated to do some rudimentary checking but the subject is deserving of further study. The work load on the operator proved high in conditions of high system activity (Brewer, Frost & Parish 1968) since not only did he have to monitor and correct system data but he had also to maintain up-to-date the computer's knowledge of station constraints which were continually changing. It also proved difficult to establish the values to be ascribed to some station constraints, particularly loading rates which tended to be limited conservatively to about 2% per minute. Under conditions of large load changes on the system some units were brought up in load transiently and then reduced as the slower loading plant picked up its load, an operation which gave rise to difficulties in the manual control of the boilers.

An attempt was made to establish the economic benefit of optimal automatic control compared with conventional operator control backed by experience and off-line studies. The problem is that the system cannot be operated under both regimes at the same time so that a direct comparison is not possible and statistical methods are necessary. The system was operated on alternate days under automatic and manual control and the cost of operation plotted as a function of load. There was considerable scatter ($\pm 6\%$) in the results due to changing system conditions, availability of plant, etc., and this was sufficient to obscure any mean difference between the two modes of operation. Moreover, if the operator allows the spare under manual control to be less than the rigid minimum specified to the computer, this would reduce the cost since the spare constraint is usually active in the optimization. Tight control of spare and security are likely to represent the greatest economic benefit of automatic control since over-provision is costly and failure to provide an adequate standard could lead to disconnexion of consumers and subsequent high loss to the economy of the society. Other paper studies have suggested that the saving in operating costs which might be achieved by on-line optimal control is about $\frac{1}{2}\%$, although the validity of this estimate must be questioned since it is doubtful whether operating costs can be determined to this accuracy.

Studies of control systems similar to that examined in the southwest experiment are being made in the U.S.A., Japan and Europe and operational experience is being gained on limited automatic control of small systems in the U.S.A. and in Germany. Studies are continuing in the U.K. of the main problems highlighted by the experiment.

Recent developments

Work has been in progress for some 3 years to define the control system on a modern generator which will ensure that it will follow the instructions of Grid Control and to determine what are the relevant operating constraints, in particular maximum loading rates for large load changes. The work is based on a coal-fired 120 MW generator of modern design at Rugeley 'A' power station and it is anticipated that the control system will be applicable to the larger (500 and 660 MW) units. A mathematical model of the dynamics has been formulated and its validity confirmed by plant tests. The problem of controlling steam pressure has been identified as due to the dynamics of the coal grinding mills and it is proposed to use a feed-forward technique to provide advanced warning of load changes. A suitable control system has been devised and tested on the model. It will be implemented on the generator and its performance evaluated under operating conditions. It is perhaps of interest to remark that modern optimal control theory appears to us to have little relevance to our problem since it is largely confined to linear systems and cannot handle constraints in a useful manner.

Laboratory studies have also been made to establish to what extent an on-line control computer can be applied to the problem of deriving a secure economic loading schedule. With suitable, usually valid, approximations the problem can be linearized and linear programming techniques applied. In the dual simplex form they lead to solutions which are acceptably fast (Brewer & Revington 1972). It is also possible to coordinate the optimization routines run in different computers in the Grid Control areas to provide overall optimization using Dantzig–Wolfe decomposition techniques (Revington 1972).

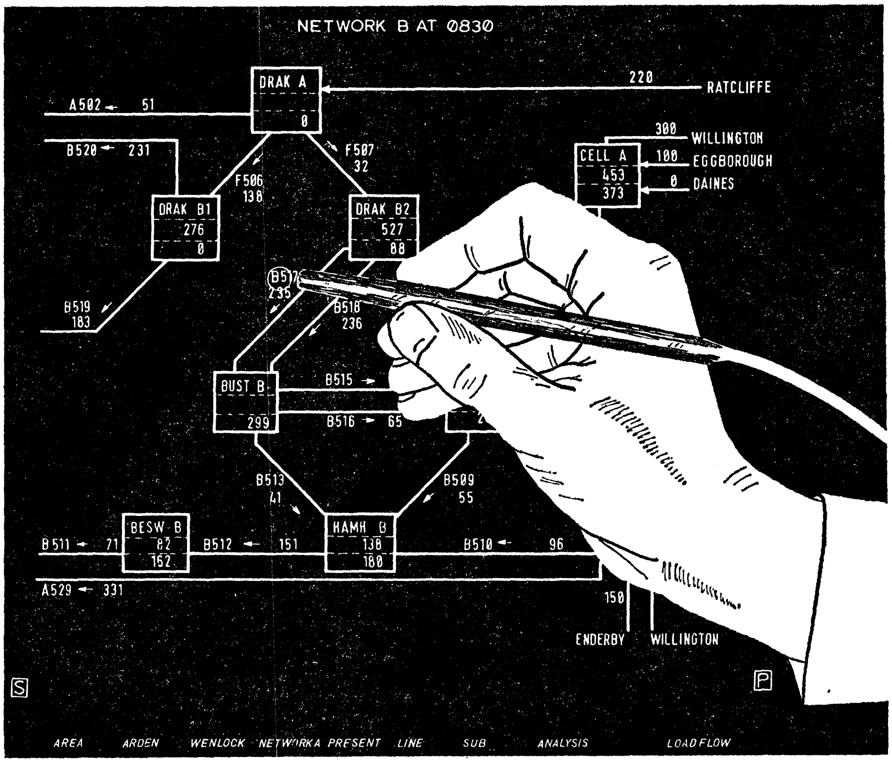

Figure 4. Operator indicating to a computer that he wishes to find the effect of taking a line out of service.

Studies of the software associated with the computer operating system, data handling and displays show that they represent 80% of the total software required so that there is a high incentive to ensure that it is as simple as possible to write, commission and modify in the light of experience. It should, therefore, be written in a high-level language and in our experience Coral 66 is appropriate. It should also be modular in form with well-defined interfaces between the individual programs. A soundly based data structure also appears essential.

Data collection still remains a problem. Accurate information is required on switch positions to determine the network configuration, on power flows in lines, generator outputs and station constraints. It could be provided by a secure, possibly duplicated, telemetry system but the cost

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would be prohibitive. A practical system is subject to errors and to interruptions. State estimation techniques are being studied and can offer some assistance, but it seems likely that there will always be a need for the operator to monitor the results to resolve ambiguities and to supply estimates of missing data based on information not available to the computer.

The operator will also require to be able to study contingencies such as the possible loss of a circuit or generator. He thus needs to interact with a display of the network in order to insert into the computer in as simple a manner as possible the data relevant to the contingency he wishes to study. Figure 4 shows an operator indicating to a computer by means of a light pen that he wishes to investigate the effect of taking a line out of service. The computer then recalculates the resulting power flows in the lines and indicates whether the loss of any other line would lead to over-loading elsewhere in the remaining network. If so the operator can ask the computer to re-schedule the generator loads to remove the insecurity as cheaply as possible. It should also be capable of defining appropriate action to be taken if there is no feasible solution, although work has still to be done in this area. Laboratory studies have shown that the calculations appropriate to an area of say 50 generators, 15 nodes and 25 lines can be carried out on an on-line computer ($2\frac{1}{2}$ μs core cycle time, 24-bit word, 16 k of core store and backing disk) in a few seconds, which is acceptable operationally. They have also shown the value of a special purpose language for display and input/output purposes (Bishop 1971).

Conclusions

It can now be predicted confidently that the technology required for automatic control of large power systems will soon be available and we can postulate a control scheme which has the following main characteristics. Each large generator is installed with an automatic control system which will enable it to follow precisely a loading profile determined at frequent (say 10 min) intervals by Grid Control with a defined capability for contributing to frequency regulation, and able to transmit to Grid Control information on its operating constraints. At the Grid Control Centre a small front-end processor collects data from the power stations and transmission substations, checks its validity perhaps with the assistance of the operator, marshalls it and transmits it to a main computer for storage and use in calculations. It also drives cathode-ray tube displays and provides the operator with an interactive capability. The main computer has the following functions:

(*a*) Accept and file data from the front-end processor.
(*b*) Predict the demands at each demand point in the area.
(*c*) Provide a secure, economic schedule for generator loading.
(*d*) Provide displays.
(*e*) Respond to the operator's requests for information and for assessment of contingencies.
(*f*) Determine the appropriate response to system faults.
(*g*) Remote control of major substations.
(*h*) Assemble data for National Control and management.

A similar installation is required at National Control to coordinate the optimization of the schedules produced by the computers at the Grid Control Centres.

The rate at which on-line control will be introduced into power systems will depend on assessments of the possible economic benefits and of the credibility of the proposals to replace manual control. Rough estimates can be given of the cost of providing an on-line control system, but it is difficult to quantify the financial benefits, particularly those of security of supply which could involve estimating the cost of a possible nationwide blackout. Credibility will be established by a continuing series of operational trials both in power stations and in Grid Control in which the operator has the power to intervene if necessary.

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